Gaia's Place in Space

The importance of orbital positions for satellites

Earth Orbits and Lagrange Points

Satellites can be launched into a number of different orbits depending on their objectives and what they are observing. Many satellites are launched into an orbit in which they circle around the Earth as it orbits the Sun (the Moon also does this!). However, this is not the only viable orbit for a satellite.

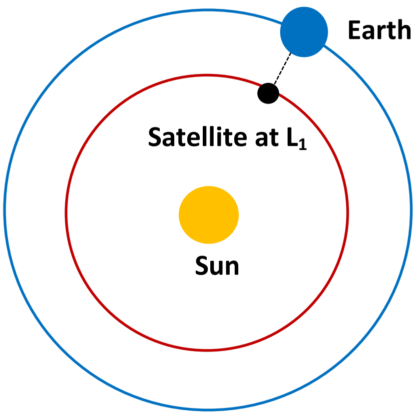
Stable orbits are also possible at special locations known as Lagrange points. At these orbital locations, satellites do not circle the Earth but rather orbit the Sun in sync with it.

The five Lagrange points, named after the astronomer and mathematician, Joseph-Louis Lagrange, are positioned at locations which are made **stable** for satellite observations due to a balance of **gravitational forces** and **orbital motions** of the Earth and the Sun.

Let's take a look at each of the points individually.

Lagrange point 1 (L_1) as shown in Figure 1 follows an orbital path that is closer to the Sun than Earth's orbit.

Figure 1 – The orbital path of a satellite located on a Lagrange point 1 orbit



Step 1: Show how the Newton (N) is the unit of measurement for the gravitational force that is calculated between two objects in the following equation:

$$F_g = \frac{G\, m_1\, m_2}{r^2}$$

Where:

F_g = force due to gravity
G = gravitational constant
m_1 = mass of central body
m_2 = mass of orbiting body
r = distance between the two bodies

Step 2: What gravitational influences would you expect an object located in the L_1 position to experience?

Step 3: What is the definition of a *centripetal force*?

Step 4: Using the equation provided and your knowledge of Newton's Law of Gravitation, for an object located *closer* to the Sun, how would you expect its orbital velocity to differ from Earth's?

$$F_c = m\,\frac{v^2}{r}$$

Where:

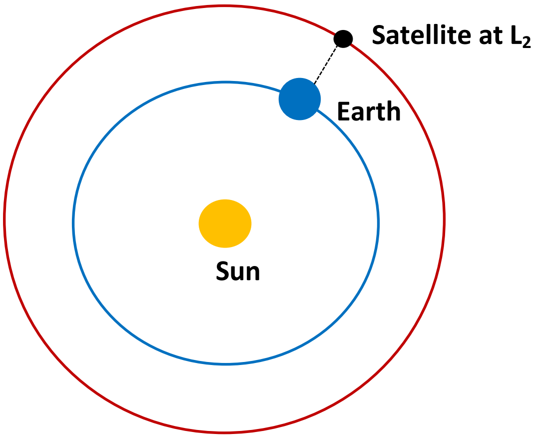
F_c = centripetal force (N)
m = mass of orbiting body (kg)
v = orbital velocity (m s^{-1})
r = distance between the two bodies (m)

However, an object on an L_1 orbit does not overtake the Earth on its orbit. L_1 is positioned at a specific distance of 1.5×10^6 km from the Earth so that the extra influence of gravity that the object should experience from the Sun is cancelled out by the gravitational pull from Earth in the opposite direction.

As a result, the object's orbital speed is **reduced** and therefore it stays in pace with the Earth.

Like L_1, **Lagrange point 2 (L_2)** is also located 1.5×10^6 km away from Earth. However, L_2 lies **further** from the Sun, placing L_2 outside Earth's orbit rather than inside like L_1. This is illustrated in Figure 2.

Figure 2 – A satellite in an orbit at the second Lagrange point, L_2.

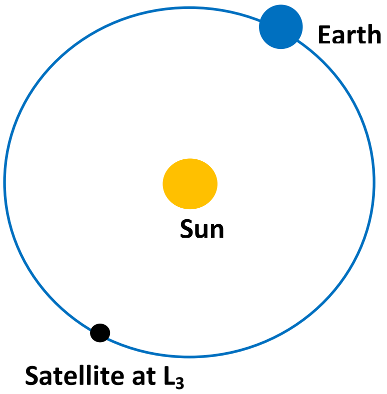


Step 5: For an object located *further away* from the Sun, how would you expect its orbital velocity to differ from Earth's?

However, again at this specific distance of 1.5×10^6 km, the gravitational influence exerted by Earth pulls the object around its orbit, **speeding it up** and allowing it to keep pace with the orbit of Earth.

Lagrange point 3 (L_3) is located at much greater distance from Earth, positioned behind the Sun, directly opposite Earth's orbital position. As shown in Figure 3, a satellite located here would follow the same orbit as the Earth just on the opposite side of the Sun.

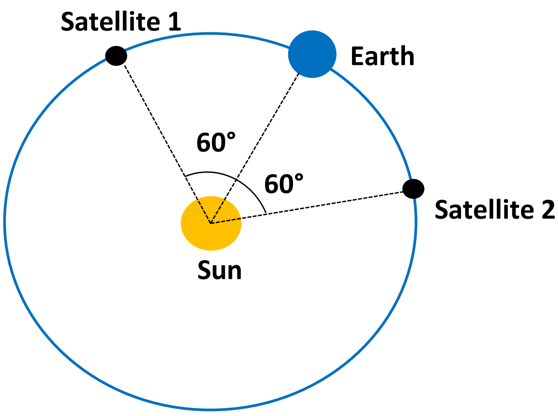
Figure 3 – The orbital path of a satellite at Lagrange point 3.



Step 6: For an object positioned at the *L_3 point*, how would you expect its orbital velocity to compare to that of Earth?

Lagrange points 4 and 5 (L_4 and L_5) are positioned at **60°** ahead (L_4) and behind (L_5) of Earth's orbit, see Figure 4.

Figure 4 – The Lagrange points 4 and 5 in relation to Earth's orbital position.



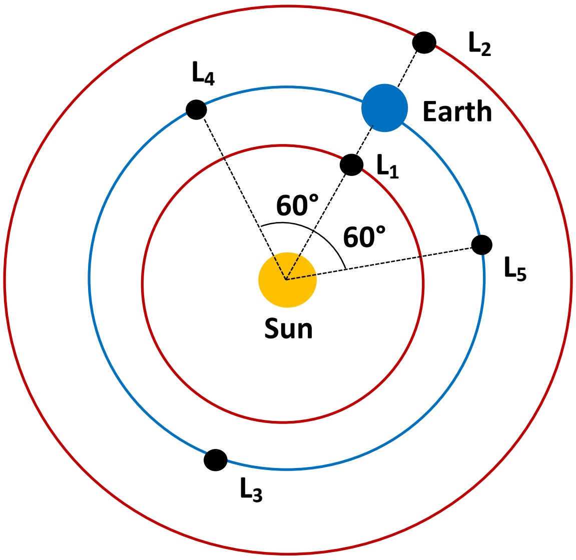
Step 7: The Earth is orbiting the Sun in an *anti-clockwise* direction in Figures 1-4. Which Lagrange points do satellites 1 and 2 correspond to in Figure 4?

To take a look at some animations of orbits at the Lagrange points, follow the link below:

http://www.esa.int/Our_Activities/Operations/What_are_Lagrange_points

Figure 5 summarises all five of the Lagrange points.

Figure 5 – The five Lagrange point orbits.



Now you are familiar with the different orbital positions known as the Lagrange points, we can apply them more specifically to Gaia.

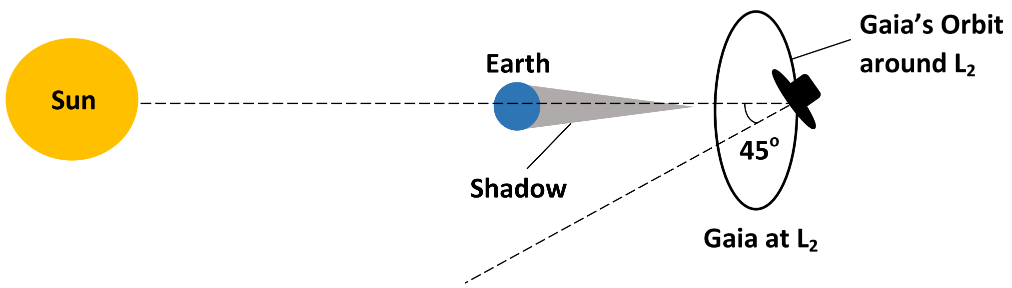
Step 8: Refer to the Introducing Gaia worksheet. Given what you know about the Gaia satellite and what its mission objectives are, which of the Lagrange points do you think would be the most suitable for Gaia's position? Justify your answer.

Step 9: You might expect Gaia to fall into the Earth's shadow due to Earth's position between Gaia and the Sun. Watch the video below to see how Gaia avoids entering the Earth's shadow:

http://sci.esa.int/gaia/53280-gaia-from-launch-to-orbit/

Here, Gaia not only keeps pace with Earth's orbit around the Sun, it also follows an **orbit around the L_2** point every **180 days**. This means that Gaia can avoid being eclipsed by Earth and going into thermal shock as a result of sudden changes in temperature. This is also illustrated in Figure 6.

Figure 6 – Gaia's orbit around L_2. This diagram is NOT to scale.



Step 10: The Moon is approximately *382,500 km* away from Earth. How many times *further away* from Earth is Gaia at the L_2 point?

Step 11: How many *km* further away from us is Gaia than the Moon?

Step 12: The Moon itself is approximately *3,476 km* in diameter. How many Moons could fit between the *Earth and Gaia* at L_2?

Step 13: Use the information provided to calculate the force of *gravitational attraction* between *Gaia and the Sun.*

Gravitational Constant = 6.67 x 10^{-11} m^3 kg^{-1} s^{-2}

Gaia – Sun Distance = 1.511 x 10^8 km

Mass of Sun = 1.99 x 10^{30} kg

Mass of Gaia = 2,030 kg

Step 14: Ignoring the additional orbit of Gaia around L_2 and accounting only for its orbital path around the Sun, what is the total distance covered by Gaia in *one orbit*? (State your answer to 4 significant figures).

Earth's Distance from the Sun = 1.496 x 10^8 km

Step 15: Gaia orbits the Sun in approximately the same time as the Earth does. Use your answer from Step 14 to calculate the *speed of Gaia's orbit around the Sun*. Give your answer in *km s^{-1}*.

Step 16: Use your answer in Step 15 and the equation below to calculate the *centripetal force* on Gaia's orbit around the Sun.

$$F_c = m\,\frac{v^2}{r}$$

Where:

F_c = centripetal force (N)
m = mass of orbiting body (kg)
v = orbital velocity (m s^{-1})
r = distance between the two bodies (m)

Step 17: How does the centripetal force you calculated in Step 16 compare with the value you obtained for the force of gravitational attraction in Step 13?

Step 18: Assume the average running speed of a human is *4.0 m s^{-1}*. How does this compare with the velocity of Gaia's orbit around the Sun?

Step 19: If you were to run the distance of Gaia's orbit around the Sun at *4.0 $m\ s^{-1}$*, how long would it take you to complete one orbit? Give your answer in *years.*

Hint: You will need to use your answer from Step 14.

Step 20: How does your answer from Step 19 compare with Gaia's mission lifetime and a human lifetime?

Step 21: What considerations need to be made when deciding on an orbit for a satellite?