# Gaia's Place in Space

***The importance of orbital positions for satellites***

## Earth Orbits and Lagrange Points

Satellites can be launched into a number of different orbits depending on their objectives and what they are observing. Many satellites are launched into an orbit in which they circle around the Earth as it orbits the Sun (the Moon also does this!). However, this is not the only viable orbit for a satellite.

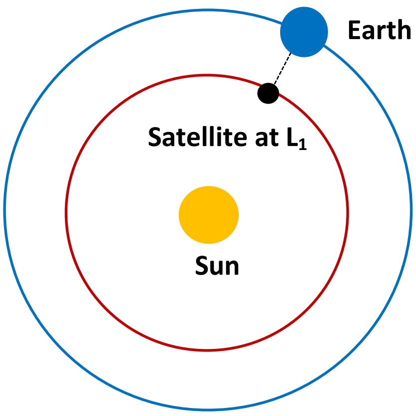
Stable orbits are also possible at special locations known as Lagrange points. At these orbital locations, satellites do not circle the Earth but rather orbit the Sun in sync with it.

The five Lagrange points, named after the astronomer and mathematician, Joseph-Louis Lagrange, are positioned at locations which are made **stable** for satellite observations due to a balance of **gravitational forces** and **orbital motions** of the Earth and the Sun.

Let's take a look at each of the points individually.

Lagrange point 1 ($L_1$) as shown in Figure 1 follows an orbital path that is closer to the Sun than Earth's orbit.

**Figure 1 – The orbital path of a satellite located on a Lagrange point 1 orbit**




**Step 1:** Show how the Newton (N) is the unit of measurement for the gravitational force that is calculated between two objects in the following equation:

$$F_g = \frac{G\,m_1\,m_2}{r^2}$$

**Where:**

**$F_g$ = force due to gravity**
**G = gravitational constant**
**$m_1$ = mass of central body**
**$m_2$ = mass of orbiting body**
**r = distance between the two bodies**

$$F_g = \frac{G\, m_1\, m_2}{r^2} = \frac{m^3\, kg^{-1}\, s^{-2}\; x\; kg\; x\; kg}{m^2}$$

Which cancels into:

$$F = kg\; m\; s^{-2} = N$$

**Step 2:** What gravitational influences would you expect an object located in the $L_1$ position to experience?

Students should recognise that this object would experience gravitational influences from both the Sun and Earth.

**Step 3:** What is the definition of a *centripetal force*?

A centripetal force is a force that causes an object to follow a circular motion, pulling the object towards the centre of this circle.

This is demonstrated in the equation:

$$F_c = m\,\frac{v^2}{r}$$

Where:

$F_c$ = centripetal force (N)
m = mass of orbiting body (kg)
v = orbital velocity (m $s^{-1}$)
r = distance between the two bodies (m)

**Step 4:** Using the equation provided and your knowledge of Newton's Law of Gravitation, for an object located *closer* to the Sun, how would you expect its orbital velocity to differ from Earth's?

$$\boldsymbol{F_c = m\,\frac{v^2}{r}}$$

**Where:**

**$F_c$ = centripetal force (N)**
**m = mass of orbiting body (kg)**
**v = orbital velocity (m $s^{-1}$)**
**r = distance between the two bodies (m)**

Students should recognise that they need an equation which relates orbital velocity (v) to distance (r) and which may include other known or constant quantities. Then they will be able to see how v varies with r. In the formula provided, $F_c$ is unknown so we need to replace it by something which is known.

Students should then recognise that the centripetal force holding a satellite or planet in orbit around the Sun is the gravitational force ($F_g$). Thus, $F_c$ can be replaced in the formula provided by the formula for the gravitational force. This gives an equation for how v relates to r with all other quantities being known or constant.

$$F_g = \frac{Gm_1m_2}{r^2}$$

Where:

$F_g$ = force due to gravity (N)
G = gravitational constant ($6.67 \times 10^{-11}$ $m^3$ $kg^{-1}$ $s^{-2}$)
$m_1$ = mass of central body (the Sun) (kg)
$m_2$ = mass of orbiting body (kg)
r = distance between the two bodies (m)

Replacing $F_c$ with the gravitational force we obtain:

$$F_g = \frac{Gm_1m_2}{r^2} = m_2\frac{v^2}{r}$$

If we then rearrange the equation to make orbital velocity (v) the subject:

$$v = \sqrt{\frac{Gm_1}{r}}$$

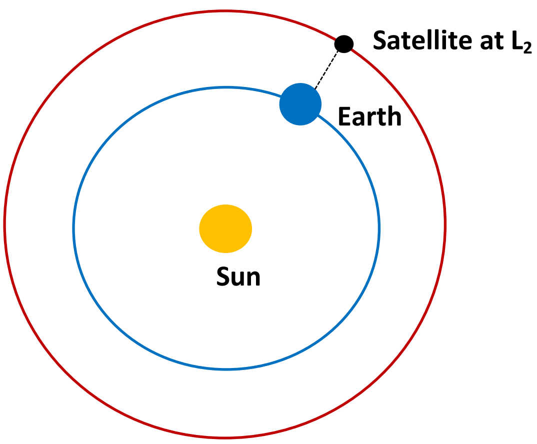
We can now see that as you get closer to the Sun and r decreases, the orbital velocity (v) of the object will increase.

However, an object on an $L_1$ orbit does not overtake the Earth on its orbit. $L_1$ is positioned at a specific distance of $1.5 \times 10^6$ km from the Earth so that the extra influence of gravity that the object should experience from the Sun is cancelled out by the gravitational pull from Earth in the opposite direction.

As a result, the object's orbital speed is **reduced** and therefore it stays in pace with the Earth.

Like $L_1$, **Lagrange point 2 ($L_2$)** is also located $1.5 \times 10^6$ km away from Earth. However, $L_2$ lies **further** from the Sun, placing $L_2$ outside Earth's orbit rather than inside like $L_1$. This is illustrated in Figure 2.

**Figure 2 – A satellite in an orbit at the second Lagrange point, $L_2$.**



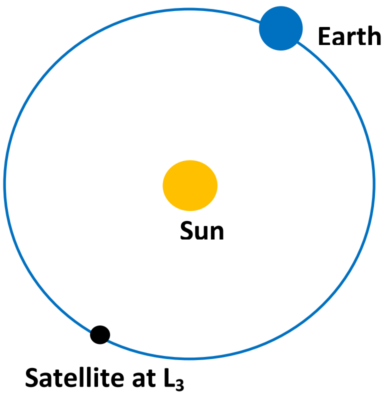
**Step 5:** For an object located ***further away*** from the Sun, how would you expect its orbital velocity to differ from Earth's?

Students should recognise that when comparing orbits of different distances, you would expect the object that lies at a distance further from the Sun to have a slower orbital velocity and therefore the closer object would overtake the object that lies further away. This is due to the weaker influence of the Sun's gravity at greater distances in accordance with the equations detailed in Step 4.

However, again at this specific distance of 1.5 x $10^6$ km, the gravitational influence exerted by Earth pulls the object around its orbit, **speeding it up** and allowing it to keep pace with the orbit of Earth.

**Lagrange point 3 ($L_3$)** is located at much greater distance from Earth, positioned behind the Sun, directly opposite Earth's orbital position. As shown in Figure 3, a satellite located here would follow the same orbit as the Earth just on the opposite side of the Sun.

**Figure 3 – The orbital path of a satellite at Lagrange point 3.**




**Step 6:** For an object positioned at the ***$L_3$ point***, how would you expect its orbital velocity to compare to that of Earth?

Again due to the equation formulated in Step 4:

$$v = \sqrt{\frac{Gm_1}{r}}$$

Where:

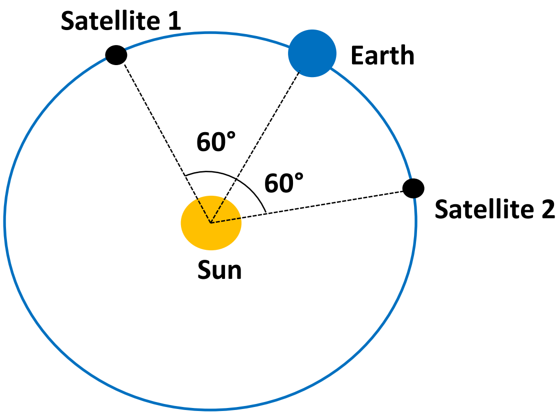
- v = orbital velocity (m $s^{-1}$)
- G = gravitational constant (6.67 x $10^{-11}$ $m^3$ $kg^{-1}$ $s^{-2}$)
- $m_1$ = mass of central body (the Sun) (kg)
- r = distance between the two bodies (m)

We see that the orbital velocity varies only with distance (r) from the Sun. (In fact, the Earth and other massive Solar System bodies will also have some gravitational influence on the satellite but this will be extremely small compared to the gravitational influence of the Sun, which is by far the dominant effect here).

Students should recognise that the object lies at the same distance from the Sun as the Earth and therefore has the same orbital velocity as the Earth. The object will continue to orbit the Sun at the same pace as the Earth but on the opposite side of the Sun.

**Lagrange points 4 and 5 ($L_4$ and $L_5$)** are positioned at **60°** ahead ($L_4$) and behind ($L_5$) of Earth's orbit, see Figure 4.

**Figure 4 – The Lagrange points 4 and 5 in relation to Earth's orbital position.**




**Step 7:** The Earth is orbiting the Sun in an *anti-clockwise* direction in Figures 1-4. Which Lagrange points do satellites 1 and 2 correspond to in Figure 4?

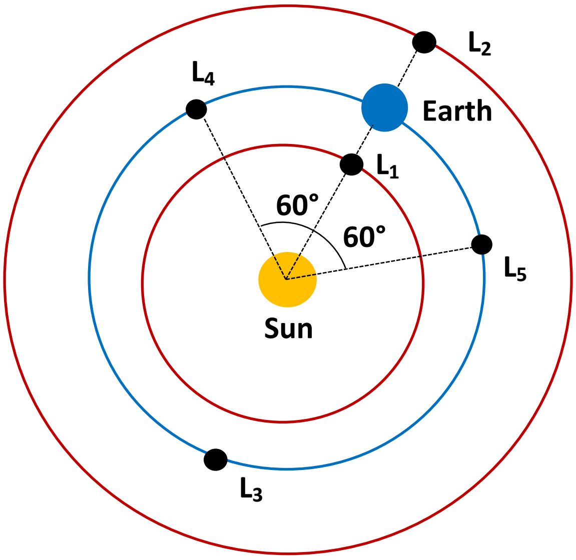
Satellite 1 is positioned at Lagrange point 4 and Satellite 2 is positioned at Lagrange point 5.

To take a look at some animations of orbits at the Lagrange points, follow the link below:

http://www.esa.int/Our_Activities/Operations/What_are_Lagrange_points

Figure 5 summarises all five of the Lagrange points.

**Figure 5 – The five Lagrange point orbits.**



Now you are familiar with the different orbital positions known as the Lagrange points, we can apply them more specifically to Gaia.

**Step 8:** Refer to the Introducing Gaia worksheet. Given what you know about the Gaia satellite and what its mission objectives are, which of the Lagrange points do you think would be the most suitable for Gaia's position? Justify your answer.

Students should recognise that $L_2$ would be the best location for Gaia. This is because the light from the Sun, Earth and Moon on one side of Gaia are blocked by the satellite's sunshield, giving Gaia an unobstructed view of the sky in the other direction. With these sources of background light blocked from getting into its instruments, Gaia is more sensitive to light from fainter distant sources and is capable of making more accurate measurements. This position also means Gaia is more easily able to stay cool as it experiences lower levels of heat from the Sun than it would on an $L_1$ orbit.

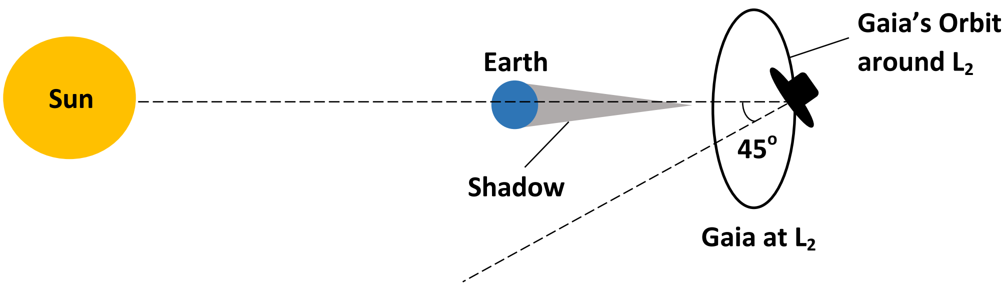
$L_1$ and $L_2$ are positioned closer to the Earth than the other Lagrange points. This stands in favour of data transmission rates.

**Step 9:** You might expect Gaia to fall into the Earth's shadow due to Earth's position between Gaia and the Sun. Watch the video below to see how Gaia avoids entering the Earth's shadow:

http://sci.esa.int/gaia/53280-gaia-from-launch-to-orbit/

Here, Gaia not only keeps pace with Earth's orbit around the Sun, it also follows an **orbit around the $L_2$** point every **180 days**. This means that Gaia can avoid being eclipsed by Earth and going into thermal shock as a result of sudden changes in temperature. This is also illustrated in Figure 6.

**Figure 6 – Gaia's orbit around $L_2$. This diagram is NOT to scale.**




**Step 10:** The Moon is approximately *382,500 km* away from Earth. How many times *further away* from Earth is Gaia at the $L_2$ point?

$$\frac{1.5\ x\ 10^6\ km - 382{,}500\ km}{382{,}500\ km} = 3.9$$

$L_2$ is 3.9 times further away from Earth than the Moon.

**Step 11:** How many *km* further away from us is Gaia than the Moon?

$$1.5\ x\ 10^6\ km - 382{,}500\ km = 1{,}100{,}000\ km$$

$L_2$ is over 1 million km further away from Earth than the Moon is.

**Step 12:** The Moon itself is approximately *3,476 km* in diameter. How many Moons could fit between the *Earth and Gaia* at $L_2$?

$$\frac{1.5\ x\ 10^6\ km}{3{,}476\ km} = 430$$

You could fit approximately 430 Moons in the space between the Earth and Gaia.

**Step 13:** Use the information provided to calculate the force of *gravitational attraction* between *Gaia and the Sun.*

**Gravitational Constant = 6.67 x $10^{-11}$ $m^3$ $kg^{-1}$ $s^{-2}$**

**Gaia – Sun Distance = 1.511 x $10^8$ km**

**Mass of Sun = 1.99 x $10^{30}$ kg**

**Mass of Gaia = 2,030 kg**

Students should again begin by converting the distance value from *km* to *m.* This is because force is measured in units of *kg m $s^{-2}$:*

$$1.511\ x\ 10^8\ km = 1.511\ x\ 10^{11}\ m$$

Then by inputting the corresponding values into Newton's Law of Gravitation we obtain:

$$F_g = \frac{G\ x\ 1.99\ x\ 10^{30}\ kg\ x\ 2{,}030\ kg}{(1.511\ x\ 10^{11}\ m)^2} = 11.8\ N$$

The force of gravitational attraction between the Sun and Gaia is approximately 12 N.

**Step 14:** Ignoring the additional orbit of Gaia around $L_2$ and accounting only for its orbital path around the Sun, what is the total distance covered by Gaia in *one orbit*? (State your answer to 4 significant figures).

**Earth's Distance from the Sun = 1.496 x $10^8$ km**

Students should recognise that here the total distance is calculated using the equation for the circumference of a circle:

$$circumference = 2\ x\ \pi\ x\ radius$$

As Gaia is positioned at $L_2$, outside the orbit of Earth, they must recognise to add Gaia's distance from Earth to the Earth's distance from the Sun:

$1.496\ x\ 10^8\ km + 1.5\ x\ 10^6\ km = 1.511\ x\ 10^8\ km$

Therefore by inputting the value calculated above into the equation for calculating the circumference, we obtain:

$orbital\ distance = 2\ x\ \pi\ x\ 1.511 x\ 10^8\ km = 9.494\ x\ 10^8\ km$

**Step 15:** Gaia orbits the Sun in approximately the same time as the Earth does. Use your answer from Step 14 to calculate the *speed of Gaia's orbit around the Sun*. Give your answer in *km $s^{-1}$*.

Students should recognise that they will need to apply the following equation:

$$speed = \frac{distance}{time}$$

They should first recognise that the Earth takes approximately 365 days to orbit the Sun and convert this into units of seconds:

$365\ days\ x\ (24\ x\ 60\ x\ 60\ s\ per\ day) = 31{,}536{,}000\ s$

Then by inputting the corresponding values into the above equation we obtain:

$$speed = \frac{9.494\ x\ 10^8\ km}{31{,}536{,}000\ s} = 30.11\ km\ s^{-1}$$

Gaia orbits the Sun at a speed of around 30 km $s^{-1}$, which is approximately the same as Earth's.

**Step 16:** Use your answer in Step 15 and the equation below to calculate the *centripetal force* on Gaia's orbit around the Sun.

$$\boldsymbol{F_c = m\,\frac{v^2}{r}}$$

**Where:**

- **$F_c$ = centripetal force (N)**
- **m = mass of orbiting body (kg)**
- **v = orbital velocity (m $s^{-1}$)**
- **r = distance between the two bodies (m)**

Students calculated Gaia's velocity in Step 15 to be 30.11 km $s^{-1}$.

Therefore by inputting the other information that is provided we obtain:

$$2{,}030\ kg\ \frac{(3.011\ x\ 10^4\ m\ s^{-1})^2}{1.511\ x\ 10^{11}\ m} = 12.2\ N$$

The centripetal force acting on Gaia's orbit around the Sun is approximately 12 N.

**Step 17:** How does the centripetal force you calculated in Step 16 compare with the value you obtained for the force of gravitational attraction in Step 13?

Students should have calculated answers which are close to each other, but with the value of the centripetal force calculated in Step 16 being higher. This is because the gravitational force on Gaia also contains contribution from the Earth, which was not included in the calculation in Step 13.

Even though the Earth is much less massive than the Sun, because it is relatively close to $L_2$ its gravitational influence will count here – in fact, as we have seen, the Earth's influence is what allows a satellite placed at $L_2$ to keep pace with the Earth even though it is on a wider orbit.

In this case, the centripetal force on Gaia is provided by a combination of the gravitational forces from the Sun and from the Earth.

**Step 18:** Assume the average running speed of a human is *4.0 m s$^{-1}$*. How does this compare with the velocity of Gaia's orbit around the Sun?

Students should have calculated the speed of Gaia's orbit around the Sun to be 30 km s$^{-1}$.

$$30\ km\ s^{-1} = 30{,}000\ m\ s^{-1}$$

Therefore:

$$\left(\frac{4\ m\ s^{-1}}{30{,}000\ m\ s^{-1}}\right) x\ 100 = 0.01\ \%$$

Humans can run at a speed equal to 0.01 % of Gaia's orbital velocity

**Step 19:** If you were to run the distance of Gaia's orbit around the Sun at *4.0 m s$^{-1}$*, how long would it take you to complete one orbit? Give your answer in *years.*

**Hint: You will need to use your answer from Step 14.**

From Step 14, students should have calculated the distance of Gaia's orbit around the Sun to be 9.494 x 10$^{8}$ km.

$$9.494\ x\ 10^{8}\ km = 9.494\ x\ 10^{11}\ m$$

The time it would take for a human to run this distance is calculated using the following equation:

$$speed = \frac{distance}{time}$$

Therefore:

$$\frac{9.494\ x\ 10^{11}\ m}{4.0\ m\ s^{-1}} = 2.4\ x\ 10^{11}\ s$$

To convert this into years we know that:

$1\ yr = (60\ x\ 60\ x\ 24\ x\ 365)\ s$

Therefore:

$$\frac{2.4\ x\ 10^{11}\ s}{(60\ x\ 60\ x\ 24\ x\ 365)\ s\ per\ yr} = 7{,}600\ yrs$$

**Step 20:** How does your answer from Step 19 compare with Gaia's mission lifetime and a human lifetime?

It would take a human approximately 7,600 years to complete just one of Gaia's orbits. Therefore, Gaia would have long finished its mission lifetime, not to mention this is far longer than any human lifetime.

**Step 21:** What considerations need to be made when deciding on an orbit for a satellite?

Answers here are at the teacher's discretion, however some of the main considerations are:

- What it is you are observing – how bright or how faint it is.
- Is it important for the satellite to be kept cool?
- Does the satellite need to be shielded from the Sun?